EXPLORATIONS

Activity 10

That's a Stretch

Objectives

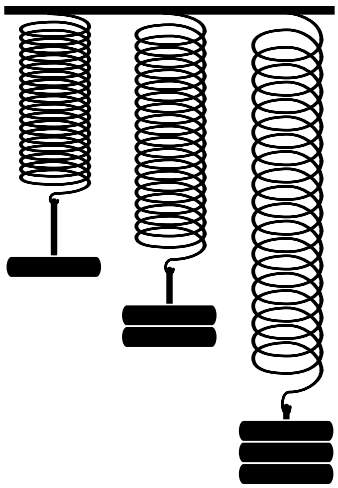
- To determine the relationship between the stretch of a spring and the number of weights in a cup suspended from the spring
- To find the *y* value of a function, given the *x* value
- To find the *x* value of a function, given the *y* value
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-73 graphing device
- Slinky® cut in half, one per group
- Small bathroom paper cups or film containers, one per group
- Marbles, pennies, or other small objects such as cubes, at least 40 per group
- Large paper clips, one per group
- Meter stick, one per group

Introduction

When you bounce a basketball, the shape of the ball temporarily changes. When you pluck a string on a guitar, the shape of the string changes. When a weight is suspended from a spring, the spring stretches. If additional weights are added, the spring stretches even more. Once the weights are removed, the spring returns to its original shape.



The basketball, the guitar string, and the spring are said to be *elastic*. If an external force is applied to an object, it creates stress within the object that causes it to become deformed. *Elasticity* is the property of a body that causes it to return to its initial size and shape after being compressed or stretched. Not all materials return to their initial state after a force is applied. These materials are said to be *inelastic*. Some examples of objects that are inelastic are clay, lead, and dough.

For many materials, the amount of stretch or compression is directly proportional to the applied force. This relationship was first expressed by British physicist Robert Hooke and is known as *Hooke's Law*.

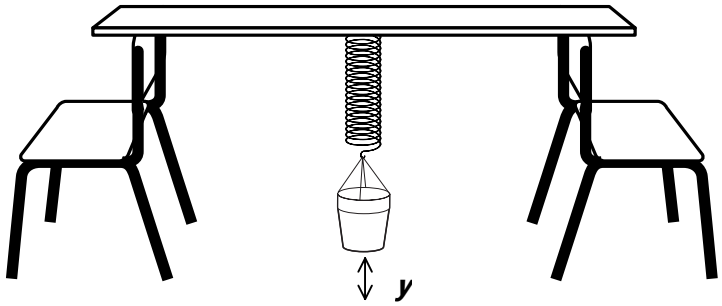
Problem

Steel is an *elastic* material. Many springs are constructed of steel. What would happen if you suspended objects from a steel spring? Would the spring stretch at a constant rate or an exponential rate?

Collecting the data — Part I

Each group of students should obtain one meter stick, a cup, a Slinky®, and 40 marbles or other small objects from your teacher.

Using a paper clip, create a handle on the cup. Hang the cup on the Slinky. Place the Slinky on the center of the meter stick. Place the meter stick across two chairs as shown in the diagram below.



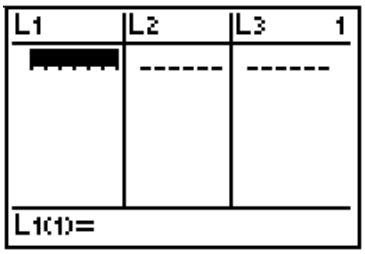
1. Measure the distance, in centimeters, from the floor to the bottom of the cup. Record the distance in the table on the **Data Collection and Analysis** page.
2. Place five of the objects that you are using in the cup. When the Slinky is stable, measure the distance from the floor to the bottom of the cup. Record the distance in the table on the **Data Collection and Analysis** page.
3. Place five additional objects in the cup. When the Slinky is stable, measure the distance from the floor to the bottom of the cup. Record the distance in the table on the **Data Collection and Analysis** page.
4. Continue placing additional objects in the cup in increments of five and measure the distance from the floor to the bottom of the cup. Record the distances in the table on the **Data Collection and Analysis** page.

Setting up the TI-73

Before starting your data collection, make sure that the TI-73 has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

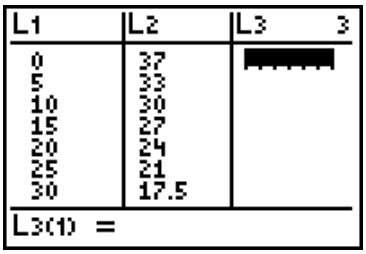
Entering the data in the TI-73

1. Press [LIST].



2. Enter the number of objects in **L1**.

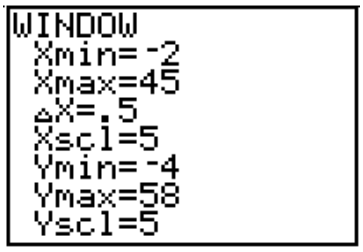
3. Enter the distance from the floor to the bottom of the cup in **L2**.



Setting up the window

1. Press [WINDOW] to set up the proper scale for the axes so that **ΔX** is **.5**.

2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.



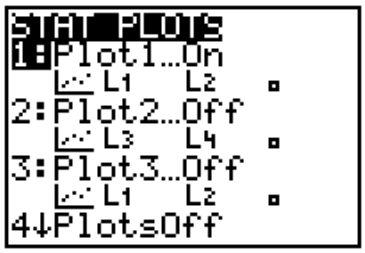
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. **Do Not Change the ΔX Value.** Set the **Xscl** to **5**.

4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.

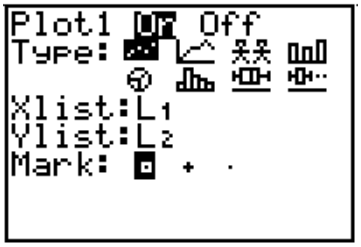
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **2**.

Graphing the data: Setting up a scatter plot

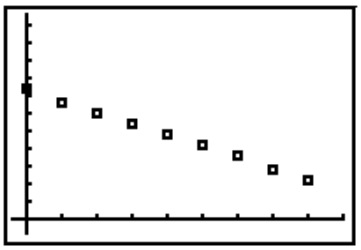
1. Press [2nd] [PLOT]. Select **1:Plot1** by pressing **1** or [ENTER].



2. Set up the plot as shown by pressing ENTER ▼ ENTER ▼ 2nd [STAT] **1:L1** ▼ 2nd [STAT] **2:L2** ▼ ENTER.



3. Press GRAPH to see the plot.



Analyzing the data

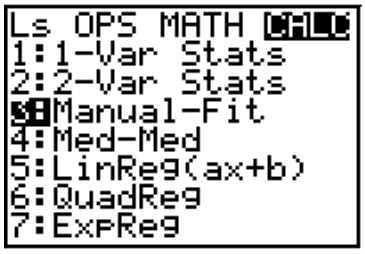
Finding a trend line

The data that you collected appears to be linear; therefore, you will find a linear equation for the line.

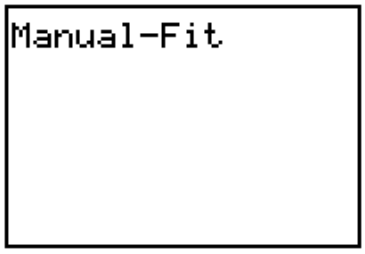
1. The *y*-intercept of a line is the point at which the line crosses the *y*-axis. The y-intercept of the trend line is the first value in **L2**. Record the *y*-intercept of the line on the **Data Collection and Analysis** page.

Find a line of best fit using the **Manual-Fit** feature on the TI-73. **Manual-Fit** allows you to fit a line to plotted data on the Graph screen manually.

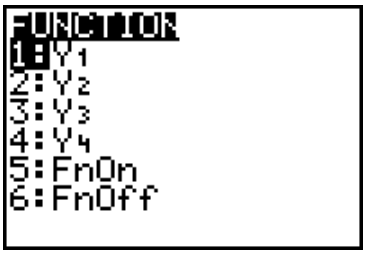
2. Press 2nd [STAT] ◀ to move the cursor to the **CALC** menu.



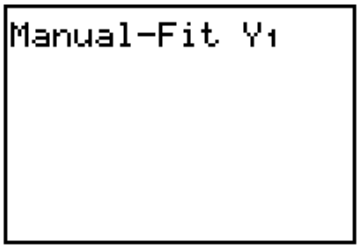
3. Select **3:Manual-Fit** by pressing **3**.



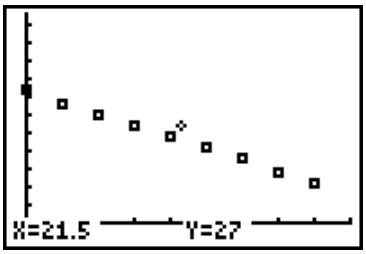
4. Press 2nd [VARS]. Select **2:Y-Vars** by pressing **2**.



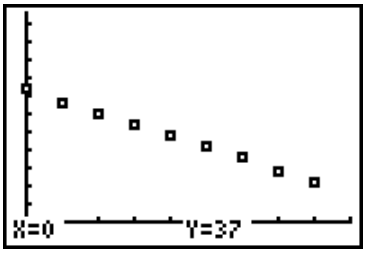
5. Select **1:Y1** by pressing **1** or ENTER.



6. Press ENTER to perform the manual fit.

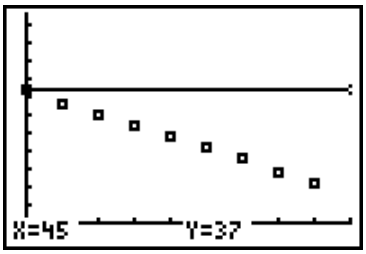


7. Use ▲ and ◀ to move the cursor to the *y*-intercept. (For this example, (0, 37).)

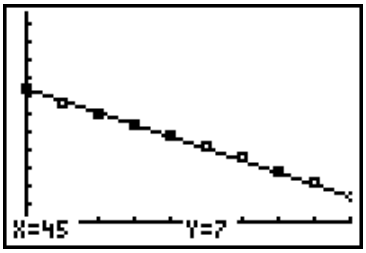


8. Press ENTER to make this point one point on the manual fit line.

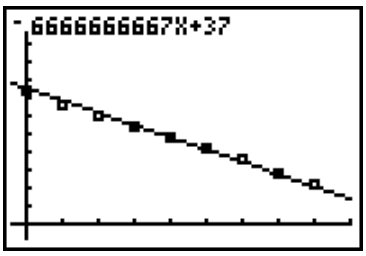
9. Press ▶ to extend a horizontal line across the screen.



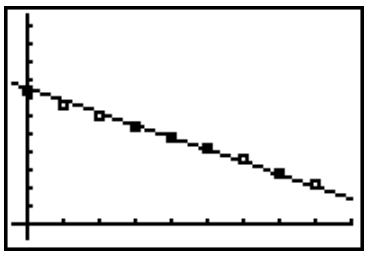
10. Press ▼ to adjust the slope of the line to match the data points.



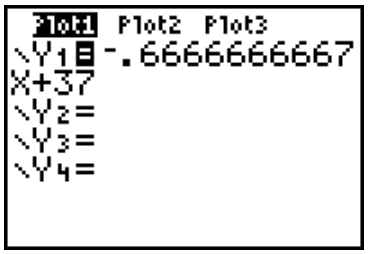
11. Press ENTER to anchor a second point on the manual fit line. (For this example, (45, 7).)



12. Use ▼ and ▲ to make adjustments to the slope. Use ◀ and ▶ to make adjustments to the y-intercept. When you have found the line you feel best represents the data, press ENTER to save the manual fit line. The equation is pasted in **Y1**.



13. Press Y= to see the equation.



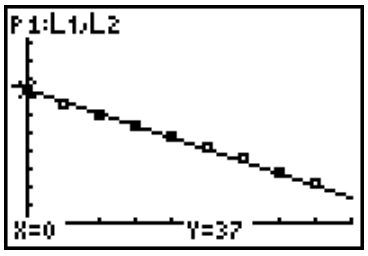
14. Record the slope and the equation of the line on the **Data Collection and Analysis** page.

Answer Part I questions 1-4 on the **Data Collection and Analysis** page.

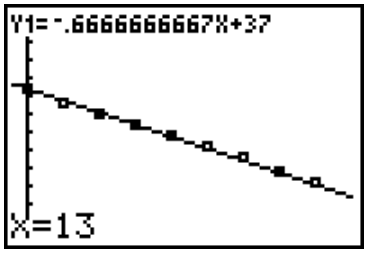
Predicting the distance

You can predict the distance the cup will be from the floor based upon the number of objects you place in the cup. Use your model to determine the distance the cup is from the floor when 13 objects are added to the cup.

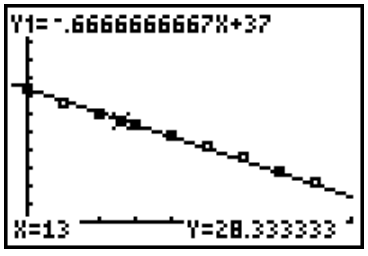
1. Press TRACE.



2. Press ▼ to get to **Y1**. Type **13** (the number of objects.)



3. Press ENTER. The *x*-value represents the number of objects, and the *y*-value represents the distance the cup is from the floor.

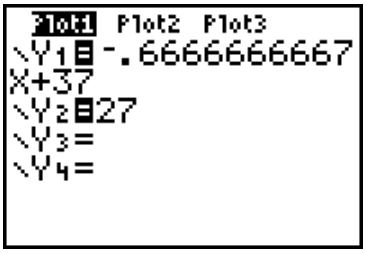


Answer Part I questions 5-6 on the **Data Collection and Analysis** page.

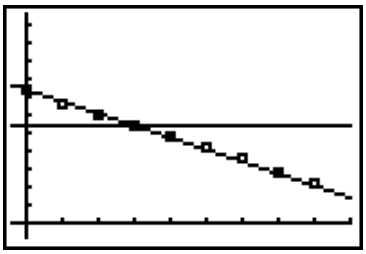
Predicting the number of objects

You can predict the number of objects in the cup based upon the distance the cup is from the floor. Use your model to determine the number of objects in the cup when the cup is 27 centimeters from the floor.

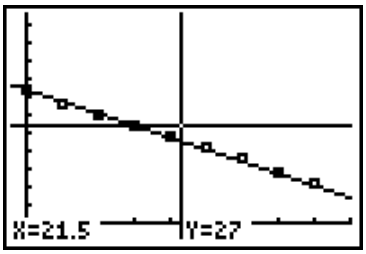
1. Press [Y=] and [▼] until you are in the first position for **Y2**. Type **27**.



2. Press [GRAPH] to see the graph of the two intersecting lines.

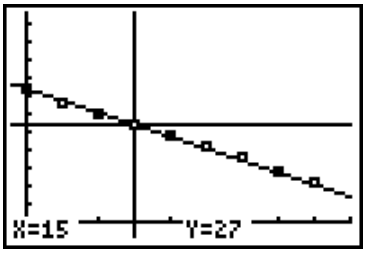


3. Draw a vertical line at the point of intersection. Press [DRAW]. Select **4:Vertical** by pressing **4**.



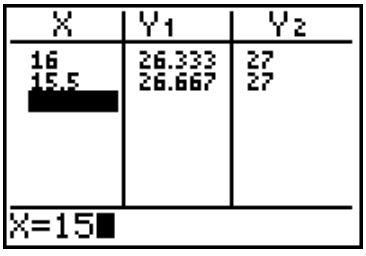
4. Use [◄] and [►] to move the vertical line until you reach the point of intersection.

***Note**: The x value is an estimate of the number of objects in the cup.*



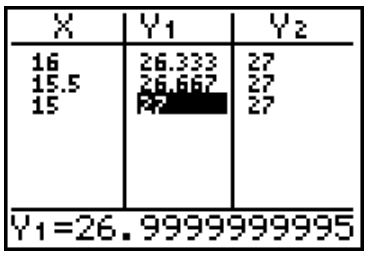
5. Use the Table to find the actual point of intersection. Press [2nd] [TBLSET]. Press [▼] [▼] [►] [ENTER] to set the Independent variable to **Ask**.

6. Press [2nd] [TABLE]. Enter *x* values and press [ENTER] until your *y* value is close to or equal to 27.



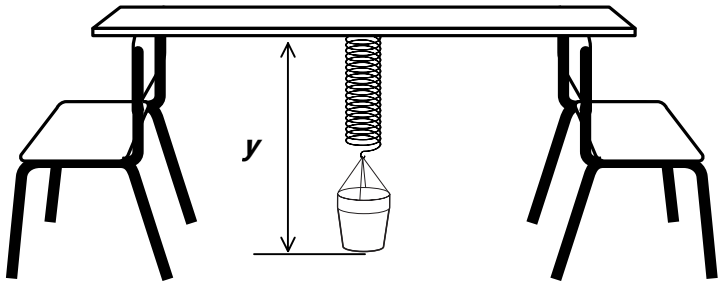
7. Press ▶ to examine the actual *y* value.

***Note**: Once you have entered seven x values, entering additional values for x will overwrite the seventh value.*



Answer Part I questions 7 - 8 on the **Data Collection and Analysis** page.

Collecting the Data — Part II



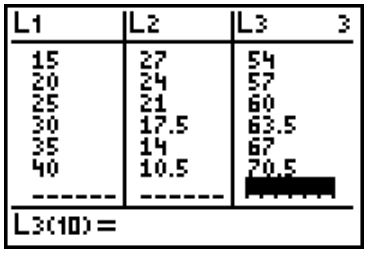
1. Remove all objects from the cup. Measure the length of the Slinky® and the cup. Place five of the objects that you are using in the cup. When the Slinky is stable, measure the length of the Slinky and the cup. Record the length in the table on the **Data Collection and Analysis** page.

2. Place five additional objects in the cup. When the Slinky is stable, measure the length of the Slinky and the cup. Record the length in the table on the **Data Collection and Analysis** page.

3. Continue placing additional objects in the cup in increments of five and measure the length of the Slinky and the cup. Record the lengths in the table on the **Data Collection and Analysis** page.

4. Measure the length of the cup with the paper clip. Record the length on the **Data Collection and Analysis** page.

5. Press LIST. Enter the length of the Slinky and the cup in **L3.**



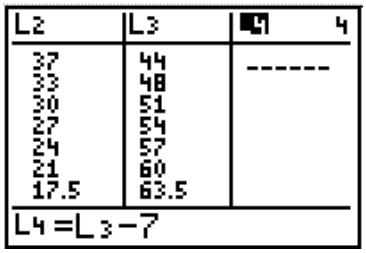
Use the following steps to calculate the length of the Slinky.

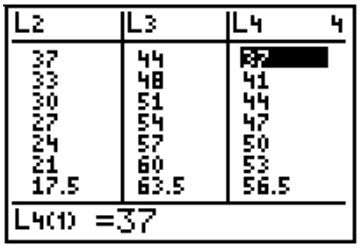
6. Press ▶ and ▲ to move the cursor to the top of **L4**.

7. Press 2nd [STAT] **3:L3** (−) the length of the cup and paper clip (recorded in Step 4.)

***Note**: For this example, the length of the cup and paper clip is 7 centimeters.*

8. Press ENTER.





9. Repeat the following Part I sections: **Setting up the window**, **Setting up a scatter plot**, and **Finding a trend line**, using the data for the number of objects (**L1**) and the length of the Slinky® and cup (**L3**). When performing the manual fit, use **Y2** instead of **Y1**. Be sure to turn off **Y1** by pressing Y= ◀ ENTER before viewing the **StatPlot**.

10. Repeat step 9 using the data for the number of objects (**L1**) and length of the Slinky (**L4**). When performing the manual fit, use **Y3** instead of **Y1**. Be sure to turn off **Y2** by pressing Y= ◀ ▼ ENTER before viewing the **StatPlot**.

Use equations **Y1**, **Y2**, and **Y3** to answer Part II questions 1 through 6 on the **Data Collection and Analysis** page.

Data Collection and Analysis

Name_____

Date _____

Activity 10: That's a Stretch

Collecting the data — Part I

Record your data from Part I in the table below.

Number of objects in cup	Distance from floor to bottom of cup (cm)
0	
5	
10	
15	
20	
25	
30	
35	
40	

Analyzing the data — Part I

The *y*-intercept is: _____.

Slope = _____ Equation of Line **Y1**: _____

Use your equation of line (**Y1**) to answer questions 1 through 8.

1. What is the *independent variable* for this activity? _____

2. What is the *dependent variable* for this activity? _____

3. Explain what the *y*-intercept represents.

_____

_____

4. Explain what the *slope* represents.

_____

_____

5. Use your equation to find the distance from the floor to the bottom of the cup if 13 objects were placed in the cup. _____

6. Actually add 13 objects to the cup and measure the distance the cup is above the floor. How does this value compare to the value predicted in question 5?

_____

_____

_____

_____

7. Using the data that you collected, determine how many objects were used if the distance measured 27 centimeters. _____

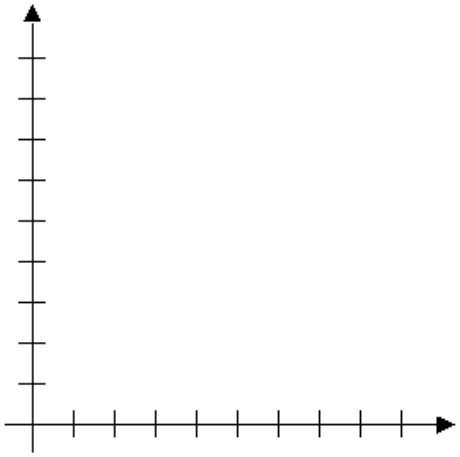
8. Jennifer did this activity with 40 pennies and Mustafa did this activity with 40 small candies (M&M's® or Skittles®). Draw a sketch of the lines produced by Jennifer and Mustafa on the same set of axes. Label the axes. Identify which line represents Jennifer's data and which line represents Mustafa's data.

a. Which person had a line with the smaller *slope*?

_____

b. Which person had a line with the greater *y*-intercept?

_____



Collecting the data — Part II

Record your data from Part II in the table below.

Number of objects in cup	Length of Slinky® and cup (cm)	Length of Slinky (cm)
0		
5		
10		
15		
20		
25		
30		
35		
40		

Analyzing the data — Part II

Length of Cup and Paper Clip = _____

For Slinky and cup:

The *y*-intercept is: _____.

Slope = _____ Equation of Line **Y2**: _____

For length of Slinky:

The *y*-intercept is: _____.

Slope = _____ Equation of Line **Y3**: _____

1. How do the *slopes* of the lines in equations **Y1**, **Y2**, and **Y3** compare?

_____

_____

2. What is the meaning of the *slope* in equations **Y2** and **Y3**?

Equation **Y2**: _____

_____

Equation **Y3**: _____

_____

3. Explain the meaning of the *y*-intercept in equations **Y2** and **Y3**.

Equation **Y2**: _____

_____

Equation **Y3**: _____

_____

4. How far would the Slinky® stretch if 13 objects were placed in the cup?

_____

***Note**: See the **Predicting the distance** section of Part I for instructions on how to do this.*

5. How many objects would it take to stretch the Slinky a distance of 75 centimeters?

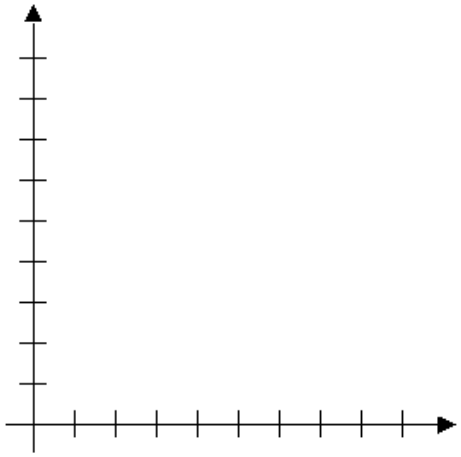
_____

***Note**: See the **Predicting the number of objects** section of Part I for instructions on how to do this.*

6. Repeat question number 6 in Part I, but this time sketch a graph for the length of the Slinky. Record you answers to **a** and **b** below.

a. _____

b. _____



Teacher Notes

Activity 10

That's a Stretch

Objectives

- To determine the relationship between the stretch of a spring and the number of weights in a cup suspended from the spring
- To find the *y* value of a function, given the *x* value
- To find the x value of a function, given the y value
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-73 graphing device
- Slinky® cut in half, one per group
- Small bathroom paper cups or film containers, one per group
- Marbles, pennies, or other small objects such as cubes, at least 40 per group
- Large paper clips, one per group
- Meter stick, one per group

Preparation

- You can suspend the meter stick across two desks or two chairs.
- You can use marbles, pennies, small cubes, or candy for objects to place in the cup.
- This activity explores both positive and negative slopes. Part II of the activity allows students to examine the *y*-intercept of a line.

Answers to Data Collection and Analysis

Collecting the data

- Sample data, Part I:

Number of objects in cup	Distance from floor to bottom of cup (cm)
0	37
5	33
10	30
15	27
20	24
25	21
30	17.5
35	14
40	10.5

- Sample data, Part II:

Number of objects in cup	Length of Slinky® and cup (cm)	Length of Slinky (cm)
0	44	37
5	48	41
10	51	44
15	54	47
20	57	50
25	60	53
30	63.5	56.5
35	67	60
40	70.5	63.5

Analyzing the data — Part I

Use your equation of line (**Y1**) to answer questions 1 through 7.

1. What is the *independent variable* for this activity?

The independent variable for this activity is number of objects.

2. What is the *dependent variable* for this activity?

The dependent variable for this activity is the distance from the floor to the bottom of the cup.

3. Explain what the *y*-intercept represents.

The y-intercept is the distance, in centimeters, from the bottom of the cup to the floor with zero objects in the cup.

4. Explain what the *slope* represents.

The slope represents the number of centimeters that the distance from the bottom of the cup to the floor changes each time an object is added to the cup.

5. Use your equation to find the distance from the floor to the bottom of the cup if 13 objects were placed in the cup.

For the sample data, the distance is 28.33 cm.

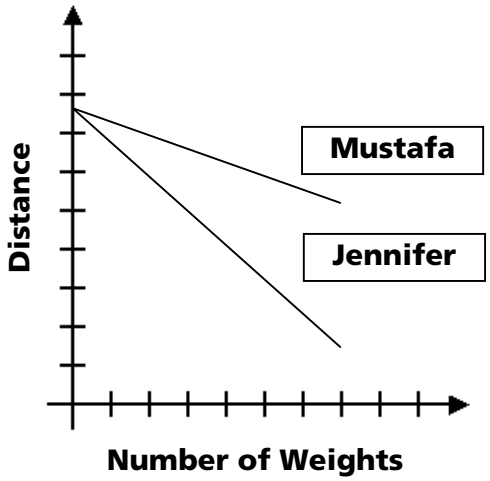
6. Actually add 13 objects to the cup and measure the distance the cup is above the floor. How does this value compare to the value predicted in question 5?

Answers will vary. The values should be close.

7. Using the data you collected, determine how many objects were used if the distance measured 27 centimeters?

The TI-73 returns a value of 15.345; however, the answer must be an integer. Therefore, the value is at least 16 objects. Check and discuss students' answers.

8. Jennifer did the activity with 40 pennies and Mustafa did the activity with 40 small candies (M&M's® or Skittles®). Draw a sketch of the lines produced by Jennifer and Mustafa on the same set of axes. Label the axes. Identify which line represents Jennifer's data and which line represents Mustafa's data.



a. Which person had a line with the smaller slope?

Mustafa

b. Which person had a line with the greater *y*-intercept?

They have the same y-intercept.

Analyzing the data — Part II

1. How do the *slopes* of the lines in equations **Y1**, **Y2**, and **Y3** compare?

The slopes of all of the lines are equal in absolute value. However, slopes of equations Y2 and Y3 are positive while the slope of equation Y1 is negative.

2. What is the meaning of the *slope* in equations **Y2** and **Y3**?

Equation **Y2**: *The number of centimeters that the length of the spring including the cup increases each time an object is added to the cup.*

Equation **Y3**: *The number of centimeters that the length of the spring increases each time an object is added to the cup.*

3. Explain the meaning of the *y*-intercept in equations **Y2** and **Y3**.

Equation **Y2**: *The initial length, in centimeters, of the spring including the cup.*

Equation **Y3**: *The initial length, in centimeters, of the spring.*

4. How far would the Slinky® stretch if 13 objects were placed in the cup?

Based upon the sample data, 52.7 cm.

5. How many objects would it take to stretch the Slinky a distance of 75 centimeters?

Based upon the sample data, 46.6 objects. Since only whole objects can be added, a reasonable value would be 47.

6. Repeat question number 6 in Part I, but this time sketch a graph for the length of Slinky®. Record your answers to **a** and **b** below.

a. *Mustafa had the graph with the smaller slope.*

b. *They have the same y-intercept.*