EXPLORATIONS

Activity 5

Objective

- To understand Euclid’s method of finding the Greatest Common Factor

Materials

- TI-73 calculator
- Student Worksheet

It’s Greek to Me

In this activity you will:

- use Euclid’s way to find the GCF (GCD) of large numbers
- investigate how this method works with the integer divide function on the TI-73

You will need to know this math vocabulary:

- greatest common divisor (GCD) or greatest common factor (GCF)
- prime number
- composite
- prime factorization

Problem

This is how you can use Euclid’s method to find the GCD of 105 and 975.

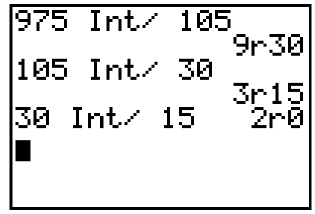
Step 1: Divide the larger number by the smaller number

Step 2: Successively divide the divisor by the previous remainder

Step 3: Continue the process until you arrive at a remainder of 0. The last divisor will be the GCD.

```
         9
  105 )975
      - 945     3
         30 )105
            - 90    2
              15 )30
                  -30
                    0
```

Activity



1. You will use the integer divide function on the calculator to find the GCD of two numbers. Set [MODE] to **Float** before you begin. Then press [2nd] [QUIT][CLEAR]. Type **9 7 5** [2nd][INT÷] **1 0 5** [ENTER]. Repeat the keystrokes to divide the previous divisor (105) by the previous remainder (15) as shown below until you arrive at a remainder of 0. The last divisor used will be the GCD.

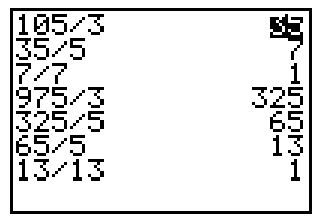
2. Recall other ways you have learned to find the GCF (GCD). Prime factorization is probably the most emphasized method. You may have learned to make a tree diagram to prime factorize a composite number. However, using divisibility rules, the calculator, and writing down the prime factors as you divide can save you some space on your paper. It is important to keep your prime factors in order and to keep yourself organized. Start dividing by the smallest prime factors first and go in increasing order until you get a quotient of 1.

3. To find GCF using prime factorization, write 105 and 975 as the product of prime factors. Find the factors (divisors) that are common to both and multiply them.

   105 = 3 x 5 x 7

   975 = 3 x 5 x 5 x 13

   GCF = 3 x 5 = 15



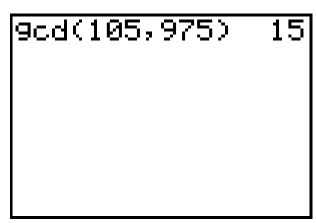
4. The first way you probably learned how to find GCF was by making lists and finding the greatest divisor common to both. You can use the prime factorization above and combinations of the factors to list the factors of 105 and 975.

   105 { 1, 3, 5 ,7, 3 x 5, 3 x 7, 5 x 7, 3 x 5 x 7 } or { 1, 3, 5, 7, **15**, 21, 35, 105}

   975 { 1, 3, 5, 13, 3x5, 5x5, 3x13, 5x13, 3x5x5, 3x5x13, 5x5x13, 3x5x5x13} or

   {1, 3, 5, 13, **15**, 25, 39, 65, 75, 195, 325, 975}

   Inspecting the lists verifies that 15 is the greatest divisor (factor).



5. Another way to check your result is to use the GCD function on the calculator.
   Press [MATH] **2:gcd(105,975** [)] [ENTER].

✎ Go to the Student Worksheet and answer the questions.

Student Worksheet

Name ______________________

Date ______________________

Activity 5

It's Greek to Me

Answer the questions about this activity.

Use Euclid's algorithm and integer divide to solve the following problems. Show your steps.

1. GCD (220, 2924)

____

____

____

2. GCD (14,595, 10,856)

____

____

____

3. Find the GCD of 1120 and 2860 two different ways: Euclid's way and Prime Factorization.

Euclid's Method	Prime Factorization
2860 [INT÷] 1120 =	2860 = 1120 = **GCF** =
1120 [INT÷] 620 =	1120 = 620 = **GCF** =
620 [INT÷] 500 =	620 = 500 = **GCF** =
500 [INT÷] 120 =	500 = 120 = **GCF** =
120 [INT÷] 20 =	120 = 20= **GCF** =

4. Compare the methods in problem 3 above. What patterns do you see?

____

____

5. Show four different ways to find GCD (105, 42).

____

____

____

____

Teacher Notes

Activity 5

It's Greek to Me

Math Strand

- Number sense
- Numeration

Materials

- TI-73 calculator
- Student Worksheets (page 41)

Students will review finding GCF using prime factorization and listing. They will investigate Euclid's algorithm of finding GCF and compare to methods already learned.

Vocabulary

Greatest Common Divisor (GCD) or Greatest Common Factor (GCF)	The greatest of the common factors or divisors of two or more numbers.
prime number	A number greater than 1 that has exactly two factors, 1 and itself.
composite	Any whole number that has more than two factors.
prime factorization	The process of writing a composite number as the product of prime factors.

Classroom Management

Students should have experienced finding GCF using the listing and prime factorization methods before doing this activity. This method works the best when trying to find the GCD of large numbers that are fairly close together.

Greatest Common Factor and Greatest Common Divisor are used interchangeably to reinforce the idea that they are synonyms.

Activity

The directions and keystrokes on the student activity pages are complete.

Answers to Student Worksheet

1. 2924 INT/ 220 = 13 R 64 ; 220 INT/ 64 = 3 R 28; 64 INT/ 28 = 2 R 8;

   28 INT/ 8 = 3 R 4; 8 INT/ 4 = 2 R 0 So GCD = 4

2. GCD (14,595, 10,856) = GCD (10,856, 3739) = GCD (3739, 3378) = GCD (3378, 361) = GCD (361, 129) = GCD (129, 103) = GCD (103, 26) = GCD (26, 25) =

   GCD (25,1)=1

3. See table below.

Euclid's Method	Prime Factorization
2860 [INT÷] 1120 = 2 r 620	2860 = 2 x 2 x 5 x 11 x 13 1120 = 2 x 2 x 2 x 2 x 2 x 5 x 7 **GCF** = 2 x 2 x 5 = 20
1120 [INT÷] 620 = 1 r 500	1120 = 2 x 2 x 2 x 2 x 2 x 5 x 7 620 = 2 x 2 x 5 x 31 **GCF** = 2 x 2 x 5 = 20
620 [INT÷] 500 = 1 r 120	620 = 2 x 2 x 5 x 31 500 =2 x 2 x 5 x 5 x 5 **GCF** = 2 x 2 x 5 = 20
500 [INT÷] 120 = 4 r 20	500 = 2 x 2 x 5 x 5 x 5 120 = 2 x 2 x 2 x 3 x 5 **GCF** = 2 x 2 x 5 = 20
120 [INT÷] 20 = 6 r 0	120 = 2 x 2 x 5 20= 2 x 2 x 2 x 3 x 5 **GCF** = 2 x 2 x 5 = 20

4. Answers will vary. In each step of Euclid's method you will get the same GCF. So you are just simplifying the problem by successively dividing. The GCF remains a factor until the end.

5. Answers will vary.

   a. The easiest and quickest way is to use the TI-73 calculator. Press [MATH], then select **2:gcd(** and type in the numbers separated by a comma. (**2:gcd(105** [,] **42**) The result is 21.

   b. A second way is to use Euclid's way and integer divide on the calculator: **105** [INT÷] **42** = **2 r 21**; **42** [INT÷] **21** = **2 r 0**. The last divisor when the remainder is 0 is the GCD. Therefore, 21 is the GCD.

c. Another method is to prime factorize, then find the divisors common to both numbers and multiply them

105 = 3 x 5 x 7

42 = 2 x 3 x 7

GCF = 3 x 7 = 21

d. A fourth method is to list the factors of both numbers and find the greatest common to both lists.

105 { 1, 3, 5, 7, 15, **21**, 35, 105}

42 { 1, 2, 3, 6, 7, 14, **21**, 42}

Going Further

1. Examine these relationships:

GCD(16,20) and GCD(16, 36)

GCD(24,36) and GCD(24, 60)

GCD(18,10) and GCD(18,28)

a. What do you notice?

b. What is the relationship between the two sets of numbers in each example? (If you add the two numbers in the 1st pair, you get the last number of the second pair. Since the first integer division of each pair has the same remainder, the GCD is also the same.)

2. Find out more information about Euclid and his mathematical contributions and report to the class.

3. Find the GCD (120,75,105) using the Euclidean Algorithm applied to two numbers at a time.

4. Have the students write the process of Euclid's algorithm in words as if they were explaining it to a student who was absent.